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ANALYSIS OF THE CONTINUOUS STELLAR TRACKING ATTITUDE REFERENCE (CSTAR) ATTITUDE RATE PROCESSOR

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The Continuous Stellar Tracking Attitude Reference (CSTAR) system is an in-house project for Space Station to provide high accuracy, drift free attitude and angular rate information for the GN&C system. The outputs of the solid state star trackers are processed to provide attitude information; rate data is then derived from the attitude. Rate derivation is based on discrete time polynomial approximation techniques. This gives simple algorithms which allow for interpolation by other users. Attitude rate is modeled as a constant with low amplitude, low frequency sinusoids superimposed.

The rate processor is parameterized to account for the effects of random errors, sample rate, data processing rate and perturbation frequency. The baseline system may be characterized as follows: the three sigma attitude accuracy is 0.01 degrees, the three sigma rate accuracy is 0.0001 degrees per second, the sample rate is 100 Hertz,the sampled signal is bandlimited to 0.5 Hertz, and the data processsing rate is 10 Hertz. The above system requires a differentiator of length 127. This will track rate perturbations of frequencies less than 0.01 Hertz with low systematic errors.

INTRODUCTION

The Orbiter uses rate gyroscopes and accelerometers as sensing devices to provide data for position and attitude control. These instruments need to be periodically updated by inertial reference information from star trackers, because their accuracy drifts with time. An alternative to this traditional system is being developed for Space Station as an in-house project of the Avionics System Division at Johnson Space Center. This project is called CSTAR for Continuous Stellar Tracking Attitude Reference. Drift free, high accuracy attitude information is supplied by using solid state star trackers to continuously track stars.

The CSTAR system requires multiple fields-of-view (f.o.v.) to provide three-axis information with minimal possibility of obscuration. The three sigma accuracy requirements for attitude and attitude rate on Space Station are presently set at 0.01 degrees and 0.0001 degrees per second respectively. This data is to be provided at a rate of 10 Hertz. The current, two f.o.v. breadboard CSTAR system is capable of meeting the three sigma attitude accuracy requirement at the 10 Hertz rate. The breadboard CSTAR uses a modifed charge injection device (CID) television camera. Attitude rate is to be derived from the attitude information to provide backup for the rate gyroscopes. The attitude rate processor at the sensor level is to meet the accuracy and sampling rate requirements in a reasonable frequency range. It is not intended to be a sophisticated, adaptive type processor which meets the very specific needs of individual users. Examples of these types of processors may be found in the references [1,2].

THEORY

An attitude rate processor must perform the operation of differentiation on the attitude data. This may be treated as a numerical analysis problem in the discrete time domain where the samples are equally spaced. Shannon's sampling theorem is of prime importance [3,4]. The system will not be able to accurately reproduce or process any data which has frequency components greater than one half of the sampling frequency. The steady state transfer function of a differentiator is given by

$H(e^{j\omega T}) = j\omega$, where T is the sample period [5].

This means that the magnitude response is proportional to frequency with a phase shift of ninety degrees for frequencies from zero to 1/2T Hertz; this response repeats periodically.

Discrete time differentiation is amenable to polynomial approximation techniques where the coefficients are derived from the data points [6]. The signal component of the attitude data is assumed to be a ramp with one or more sinusoids superimposed on it. The ramp slope is proportional to the dc component of the rate; the sinusoids are assumed to be of relatively low frequency. The ramp portion of the signal can be correctly differentiated by an approximation of order one or greater. A sinusoid cannot be represented by a finite sum of polynomials. No matter how high the order of the approximation, it will not exactly reproduce the derivative of a sinusoid.

Lagrange approximation techniques yield a curve fit in which the order of the approximation is equal to the number of data points used for the fit [7]. An example of this type of fit is given by the Stirling approximation for the derivative [8]. A sixth order approximation yields the following transfer function:

$$H(z) = (z^3 - 9 z^2 + 45 z - 45 z^{-1} + 9 z^{-2} - z^{-3})/60$$

This transfer function may be made causal by multiplying by z^{-3}. A plot of the magnitude response of the Stirling differentiator and the ideal differentiator is shown in Figure 1. The frequency in Hertz is given by N/(1024T) where N is the frequency value on the plot. The Stirling differentiator closely approximates the ideal response in the lower frequency range; higher frequencies are attenuated. This should not be of concern, because the sample rate can always be chosen so that the frequencies of interest are not significantly attenuated. From a noise standpoint, the attenuation of higher frequencies may be desirable.

In practice, numerical differentiation is a more difficult operation than numerical integration [9,10]. The data to be differentiated are made up of a signal component and a noise component. The noise is assumed to be zero mean, uncorrelated, white Gaussian noise. This assumption is confirmed by analysis of the experimental data obtained from the CSTAR breadboard system. When this random noise is differentiated, the derived rate can have a very large noise component added to it. The Stirling differentiator shown in Figure 1 does not sufficiently attenuate the random noise to meet the rate accuracy requirements. A least squares type of approximation is more appropriate for this application [11].

In a least squares curve fit, the order of the approximation, m, is equal to or less than the number of data points to be fit, L. The length of the digital filter which realizes the approximation is L. In this application, the spacing between the data points is uniform and is equal to the sample period, T. The point at which the approximation is made is also a variable. For example, it may be placed in the middle of the data points or at the end. Errors in the approximation are due to both the random errors in the original signal, and the systematic errors introduced by the approximation itself. The above parameters affect how the random errors are reduced, and how large the systematic errors are. Random errors are decreased by the following:

1). Increasing the filter length, L.

2). Increasing the sample period, T.

3). Using a lower order approximation, m.

4). Smoothing to the center.

The systematic errors are reduced by the following:

1). Decreasing the filter length for a given sample period.

2). Decreasing the sample period for a given length.

3). Using a higher order approximation.

4). Smoothing to the center.

It can be seen from the above statements that, except for smoothing to the center, the selection of parameters to reduce random errors is in direct conflict with the need to choose parameters to decrease the systematic errors. Although smoothing to the center is desirable, it is not possible in a causal system without introducing time delay.

The variance reduction factor (VRF) for the approximation of the derivative is given by the following:

$$\text{VRF} = K/(\,T^2 L^3\,) \quad \text{for large values of L.}$$

K is a constant which depends on the order of the approximation and the point at which the data is smoothed. If L is not large, the exact expression will give a VRF which is smaller than the one obtained from the above equation. The systematic errors will be considered in relation to the steady state frequency response of the digital filter which implements the differentiator. This type of treatment lends itself to the analysis of signals which are other than sinusoids. A Fourier decomposition of an arbitrary function will yield its sinusoidal components. Superposition can be used to obtain the systematic error for the function.

The derivation of the coefficients in terms of the data points for a second order least squares curve fit, smoothed to the center follows: The function evaluated at the jth point from the center point is

$$f(j) = p_0 + p_1 j + p_2 j^2$$

Differentiating this gives

$$d[f(j)]/dj = p_1 + 2p_2 j$$

The coefficients may be calculated from the following set of simultaneous equations:

$$\begin{aligned}
p_0 \sum_{k=-N}^{N} 1 + p_1 \sum_{k=-N}^{N} k + p_2 \sum_{k=-N}^{N} k^2 &= \sum_{k=-N}^{N} x_{K-k} \\
p_0 \sum_{k=-N}^{N} k + p_1 \sum_{k=-N}^{N} k^2 + p_2 \sum_{k=-N}^{N} k^3 &= \sum_{k=-N}^{N} k\, x_{K-k} \\
p_0 \sum_{k=-N}^{N} k^2 + p_1 \sum_{k=-N}^{N} k^3 + p_2 \sum_{k=-N}^{N} k^4 &= \sum_{k=-N}^{N} k^2 x_{K-k}
\end{aligned}$$

where $(2N + 1) = L$; N is an integer. Let $S_m(N) = \sum_{k=0}^{N} k^m$, the simultaneous equations reduce to

$$\begin{bmatrix} L & 0 & 2S_2(N) \\ 0 & 2S_2(N) & 0 \\ 2S_2(N) & 0 & 2S_4(N) \end{bmatrix} \begin{bmatrix} p_0 \\ p_1 \\ p_2 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & \cdots & 1 & 1 & 1 & \cdots & 1 & 1 \\ N & N-1 & \cdots & 1 & 0 & 1 & \cdots & -(N-1) & -N \\ N^2 & (N-1)^2 & \cdots & 1 & 0 & 1 & \cdots & (N-1)^2 & N^2 \end{bmatrix} \begin{bmatrix} x_{k+N} \\ \vdots \\ x_k \\ \vdots \\ x_{k-N} \end{bmatrix}$$

Expressions for S_m are given by

$$S_1 = N(N + 1)/2$$

$$S_2 = N(N + 1)(2N + 1)/6$$

$$S_3 = L^2(L + 1)^2/4$$

$$S_4 = L(L + 1)(2L + 1)(3L^2 + 3L + 1)/30$$

The simultaneous equations give simple solutions for the polynomial coefficients in terms of the data points. The approximation of the derivative at the endpoint is obtained by setting j = N. The same transfer function is obtained if the simultaneous equations are set up to smooth to the endpoint directly. Such an evaluation involves the inversion of a 3x3 matrix, which is not necessary in the above derivation. It is also useful to have all of the coefficients availible so that users may obtain rate estimates for times which are not sample points.

The sampling rate and the rate at which the data points are processed need not be the same [12]. Time decimation requires that the sampled signal is bandlimited to prevent aliasing at the processing rate. Bandlimiting the original signal will eliminate some of the high frequency components. This reduces the standard deviation of the random noise. A shorter length filter is then required to achieve the same overall variance reduction. The systematic errors may thereby be improved.

RESULTS

It is desirable to look at variance reduction in a normalized fashion. All of the calculations and graphs are done in terms of the ratio of the rate standard deviation, σ_R, to the position standard deviation, σ_A. The ratio will be denoted by R for convenience. For the Space Station requirements specified, R = 0.01. Figure 2 shows the differentiator length as a function of the sample period for a second order approximation smoothed to the endpoint with L determined by the equation below.

$$L = [192/(R^2T^2)]^{1/2}$$

A differentiator of length 577 is required for a sample rate of 10 Hertz and R = 0.01. For a sample rate of 1 Hertz and R = 0.25, the differentiator length required is reduced to 15.

The magnitude response of a second order, length 15 differentiator smoothed to the endpoint is presented in Figure 3, along with the ideal response. The frequency in Hertz is given by N/(1024T) where N is the frequency value on the plot. The magnitude response agrees well with the ideal at very low frequencies; frequencies above this range are over emphasized in comparison to the ideal response. High frequencies are attenuated; this is consistent with the desire to reduce the variance of the Gaussian noise. A sine wave with amplitude of one and frequency of 1/5400 Hertz (this is roughly the rate at which the Space Station will orbit the Earth) was differentiated by the above filter. A plot of the differentiated sine wave, both ideal and filtered, is given in Figure 4. The agreement between the ideal and the filtered waveforms is excellent with only the first 14 samples showing any major deviation.

The systematic error for a given differentiator was related to the difference between the magnitude response of the ideal transfer function and that of the filter. This treatment ignores the phase response which makes it inexact. At low frequencies, the phase shift of the filter from ninety degrees is very small. This justifies the use of only the magnitude response at low frequencies. At high frequencies, the phase shift is large and adds significantly to the systematic error. The magnitude error at these frequencies is also very large, and operation at these frequencies should be avoided on that basis alone. Plots of the systematic error versus frequency for second order differentiators smoothed to the end point are given in Figures 5 and 6. In each case, the sampling rate and the processing rate are the same: 10 Hertz and 1 Hertz respectively.

If the data is sampled at one rate, and then processed at another rate, the sampled signal must be bandlimited so that there is no aliasing at the lower rate. Bandlimiting also produces the desirable effect of reducing the standard deviation of the Gaussian noise. Fifth order digital Butterworth filters with prewarping are used for bandlimiting [13,14]. Table 1 gives the standard deviation, after bandlimiting, of a set of 10,000 normally distributed randoms numbers with original standard deviation of 1. Reducing the standard deviation of the random noise via bandlimiting reduces the length of the differentiator needed to meet a certain set of specifications. This, in turn, improves the systematic errors.

Several schemes for sampling and processing at differents rates are presented in Table 2. The systematic errors for each of these processing schemes are given in Figures 7 through 10. In all of the graphs, T represents the sample period, and τ represents the period at which the data is processed. Figures 11 through 16 present a comparison of the systematic errors of the appropriate schemes for various processing rates and values of R.

CONCLUSIONS

The above analysis shows that it is difficult to achieve differentiators which have the desired data bandwidth, reduce random errors, and accuratly process signals with other than very low frequency components. The length 577 differentiator which meets the current Space Station requirements for accuracy and bandwidth will have systematic errors comparable to the random errors for frequencies greater than 0.001 Hertz; this is assuming a sine wave of amplitude one. For bandlimiting R = 0.01. If R is increased to 0.1 in scheme D, the usable frequency range edges up to about 0.01 Hertz. This decrease in the value of R could represent a relaxation of the Space Station requirements and/or the improvement of the CSTAR attitude processor accuracy. Bandlimiting below 0.5 Hertz should provide greater variance reduction, and, therefore, shorter length differentiators with lower systematic errors.

The digital filters needed for bandlimiting and differentiation can be implemented in a straight forward manner with currently available processors. Using Texas Instruments' TMS320 family, the number of machine cylces per iteration of the filter is about equal to the length of the differentiator (a nonrecursive filter), and twice the order of the Butterworth lowpass filter (a recursive structure). Some overhead must be added for inputting and outputting data, setting up registers, etc. The TMS32025C has a cycle time of 100 nanoseconds with very low power consumption. Other processors should give comparable results.

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TABLE 1

REDUCTION OF THE STANDARD DEVIATION WITH BANDLIMITING

$\omega_c T/2\pi$	SIGMA
1×10^{-3}	3.46×10^{-2}
2×10^{-3}	5.66×10^{-2}
5×10^{-3}	1.01×10^{-1}
1×10^{-2}	1.43×10^{-1}
2×10^{-2}	2.00×10^{-1}
5×10^{-2}	3.14×10^{-1}
1×10^{-1}	4.45×10^{-1}
2×10^{-1}	6.27×10^{-1}

TABLE 2

SCHEMES FOR BANDLIMITING AT DIFFERENT SAMPLING AND PROCESSING RATES

SCHEME	CUTOFF FREQUENCY	SAMPLING RATE	PROCESSING RATE
A	5 HERTZ	100 HERTZ	10 HERTZ
B	0.5 HERTZ	100 HERTZ	1 HERTZ
C	0.5 HERTZ	10 HERTZ	1 HERTZ
D	0.5 HERTZ	100 HERTZ	10 HERTZ

FIGURE 1

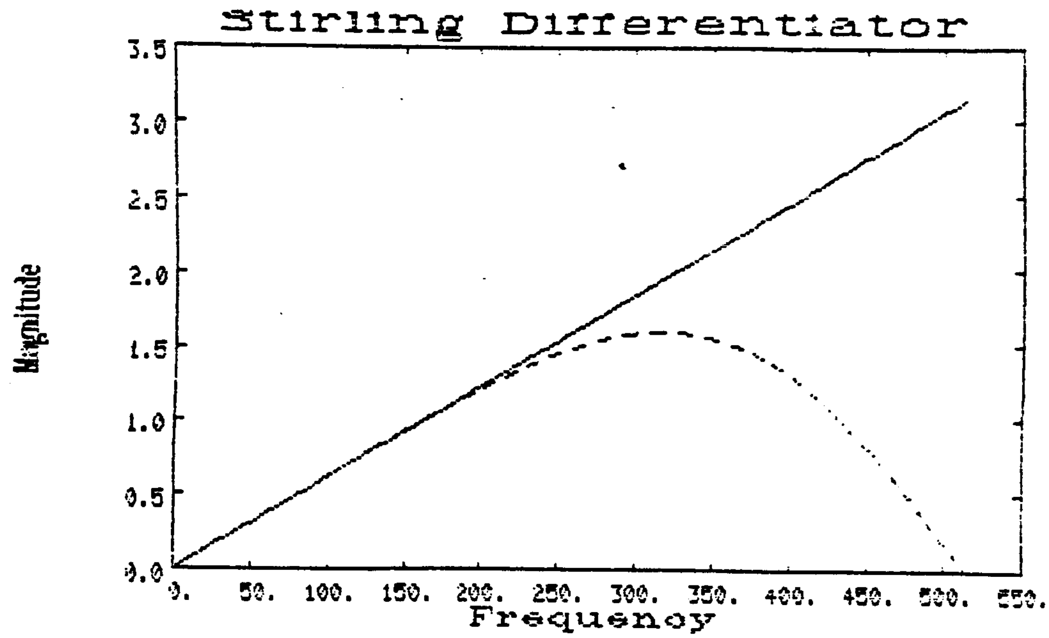
Stirling Differentiator
Magnitude
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.
50.
100.
150.
200.
250.
300.
350.
400.
450.
500.
550.
Frequency

FIGURE 2
SECOND ORDER APPROXIMATION

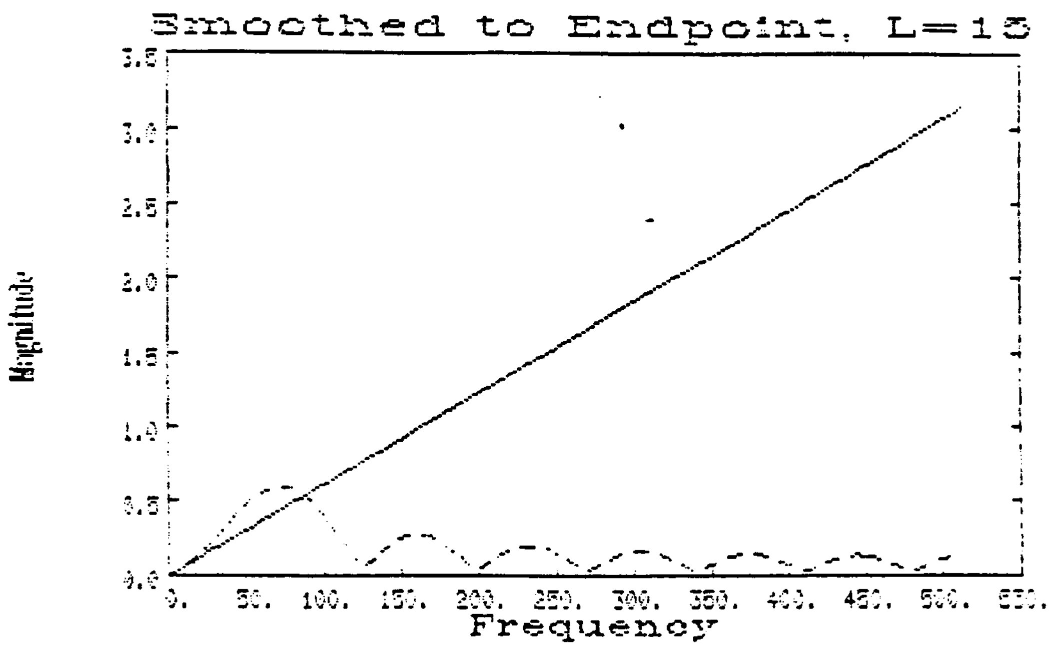

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FIGURE 3
SECOND ORDER APPROXIMATION, L=15
SMOOTHED TO THE ENDPOINT

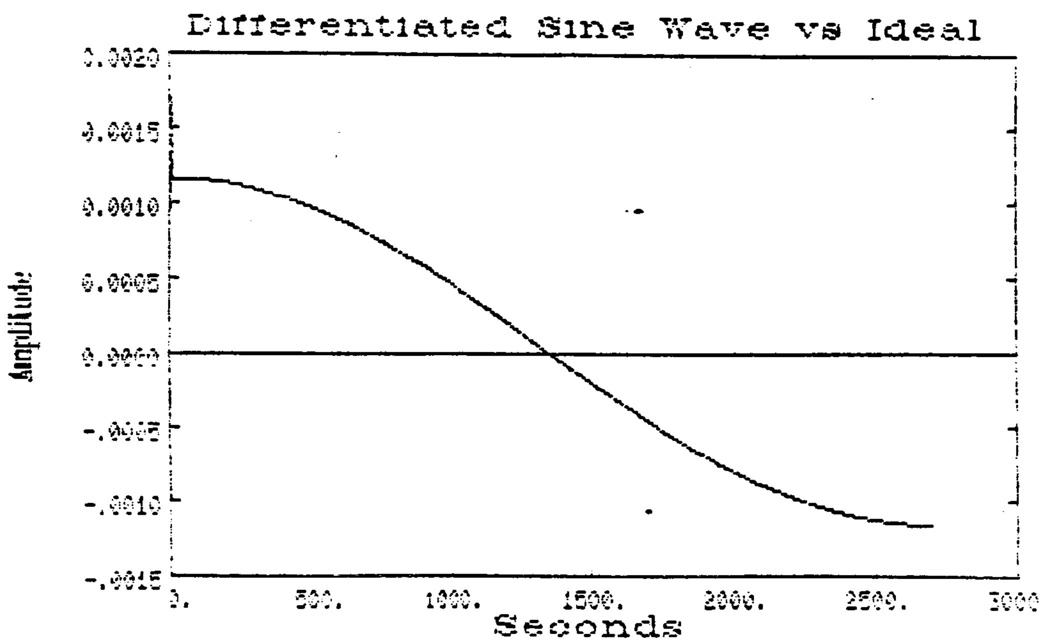
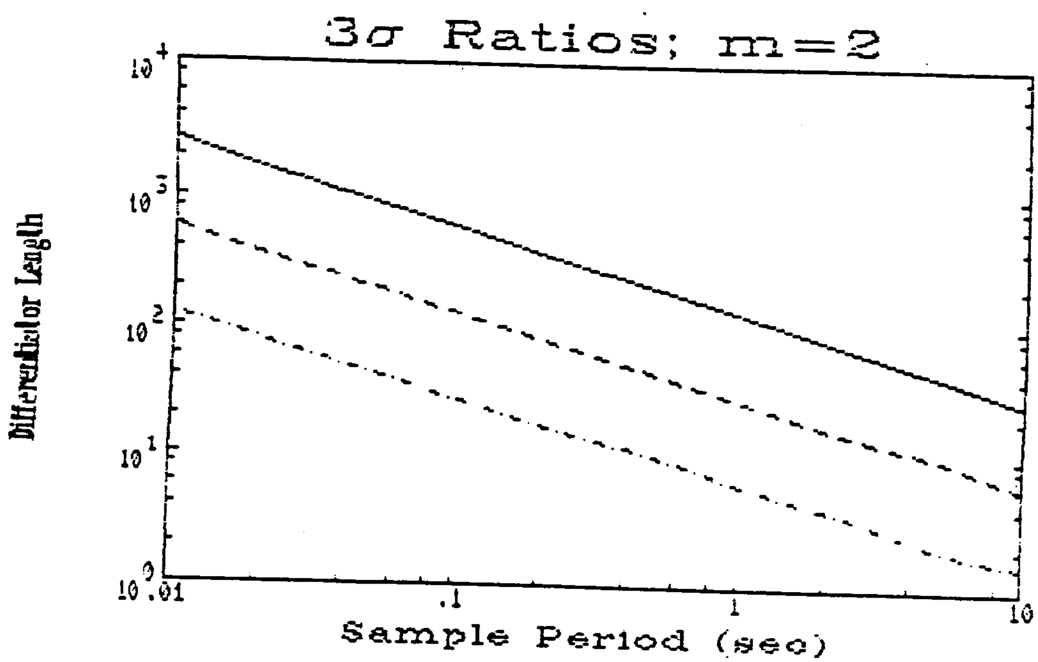


FIGURE 4
SMOOTHED TO THE ENDPOINT



Legend

Linetype	R
Solid	0.01
Dashed	0.10
Dotdash	1.00

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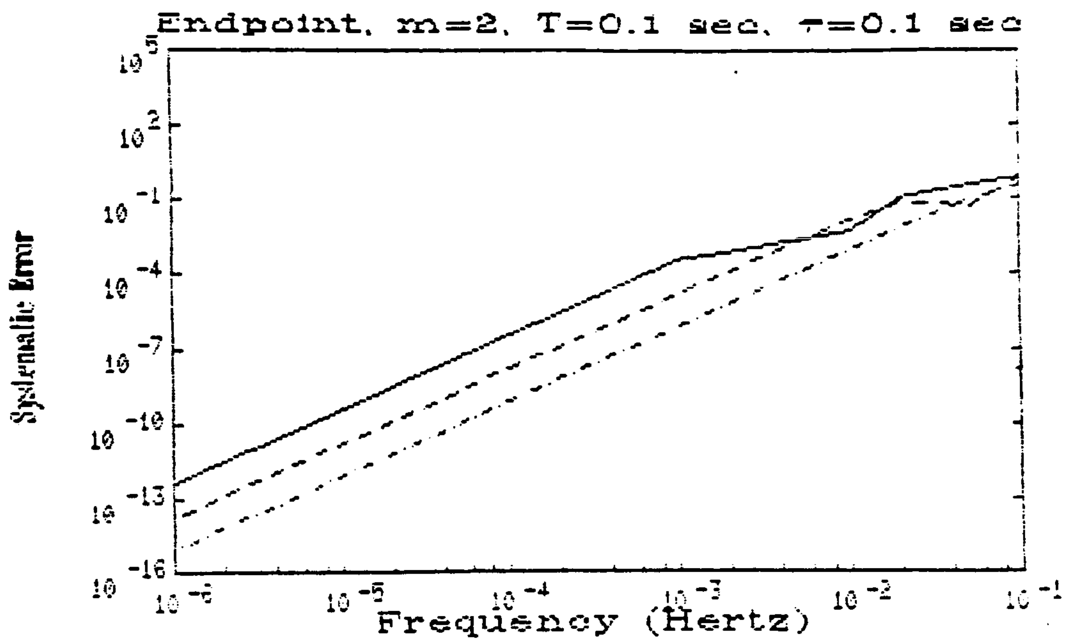
FIGURE 5



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Linetype	R
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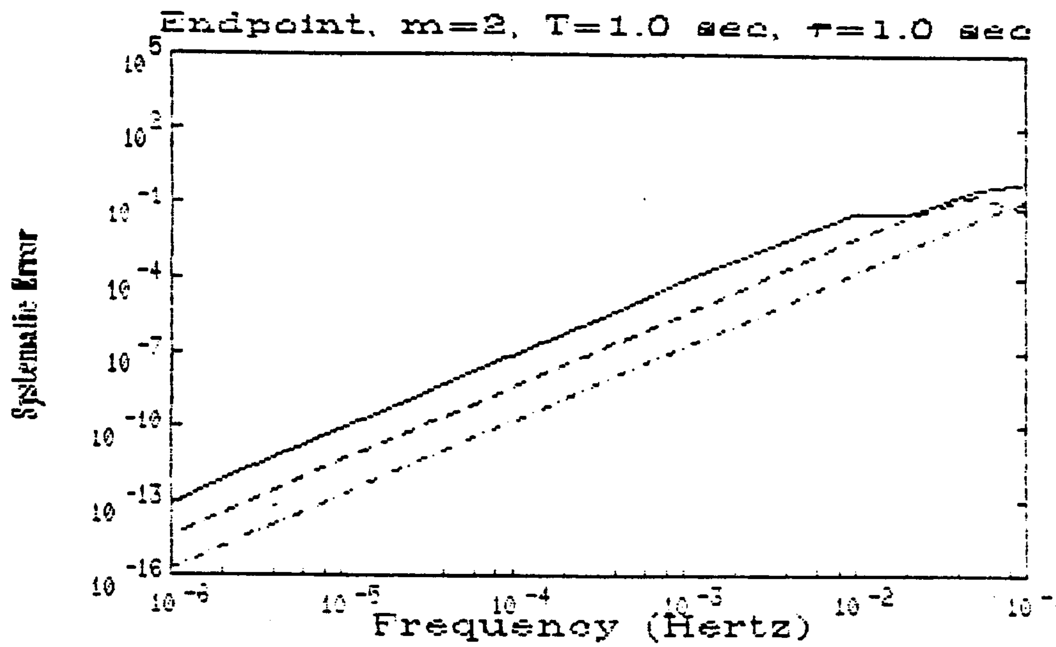
Dotdash	1.00

FIGURE 6



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Linetype	R
Solid	0.01
Dashed	0.10
Dotdash	1.00

FIGURE 7
SCHEME A

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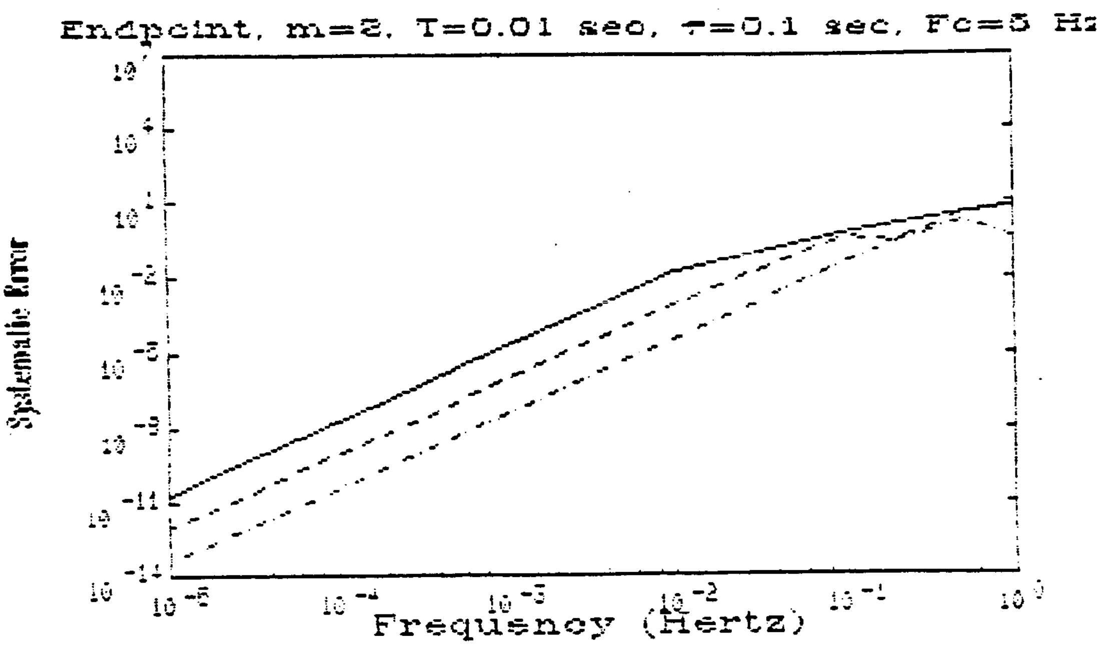

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Linetype	R
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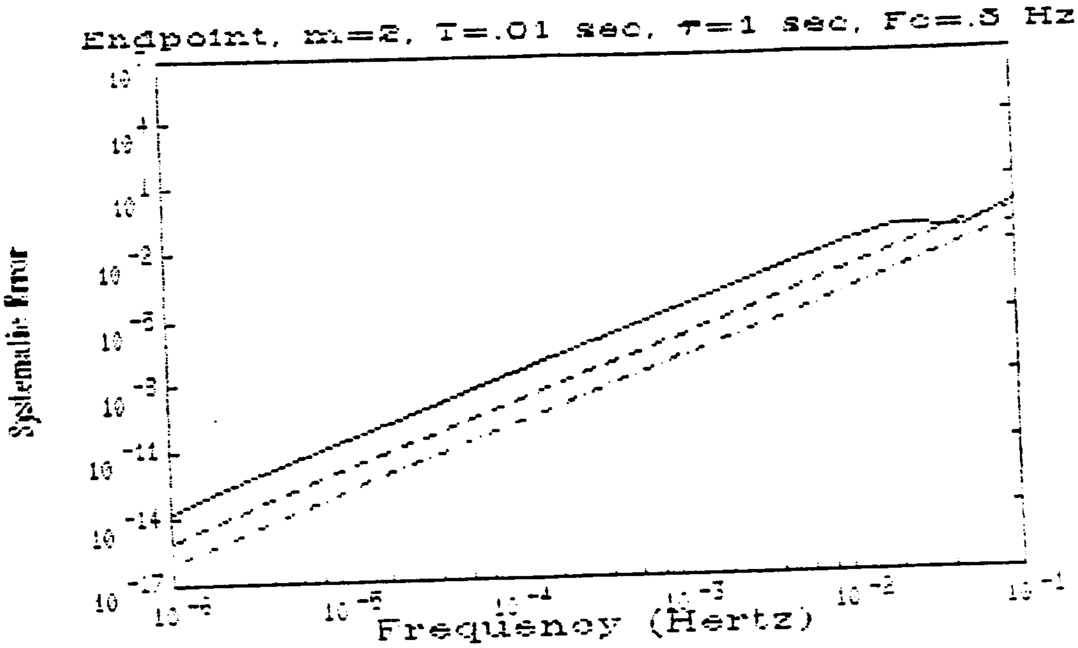
Dotdash	1.00

FIGURE 8
SCHEME B



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Linetype	R
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Dotdash	1.00

FIGURE 9
SCHEME C

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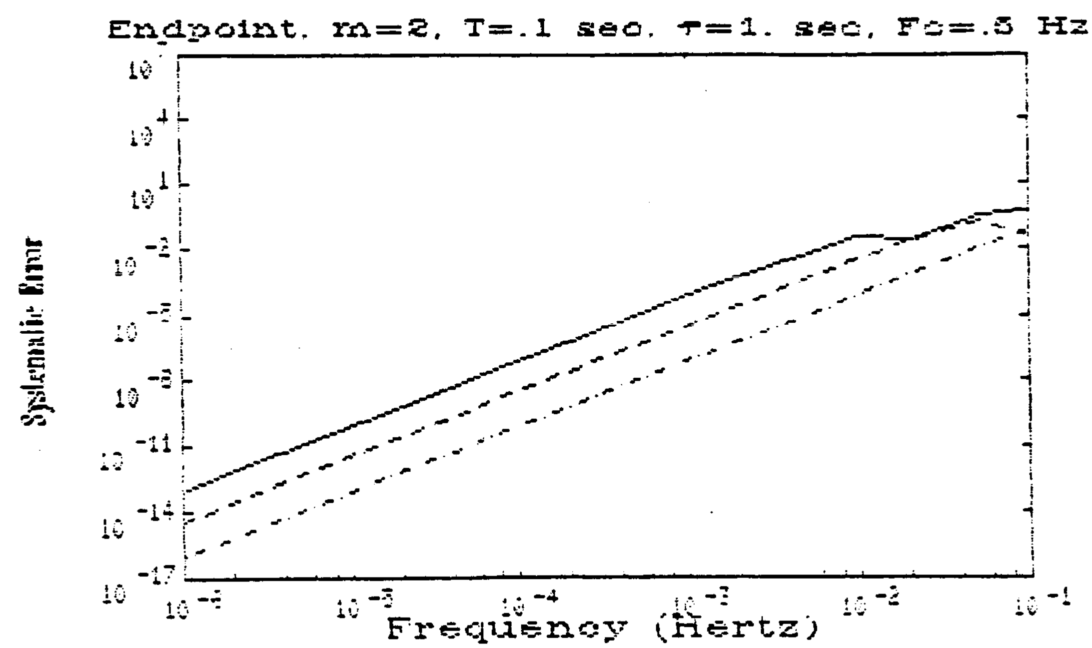

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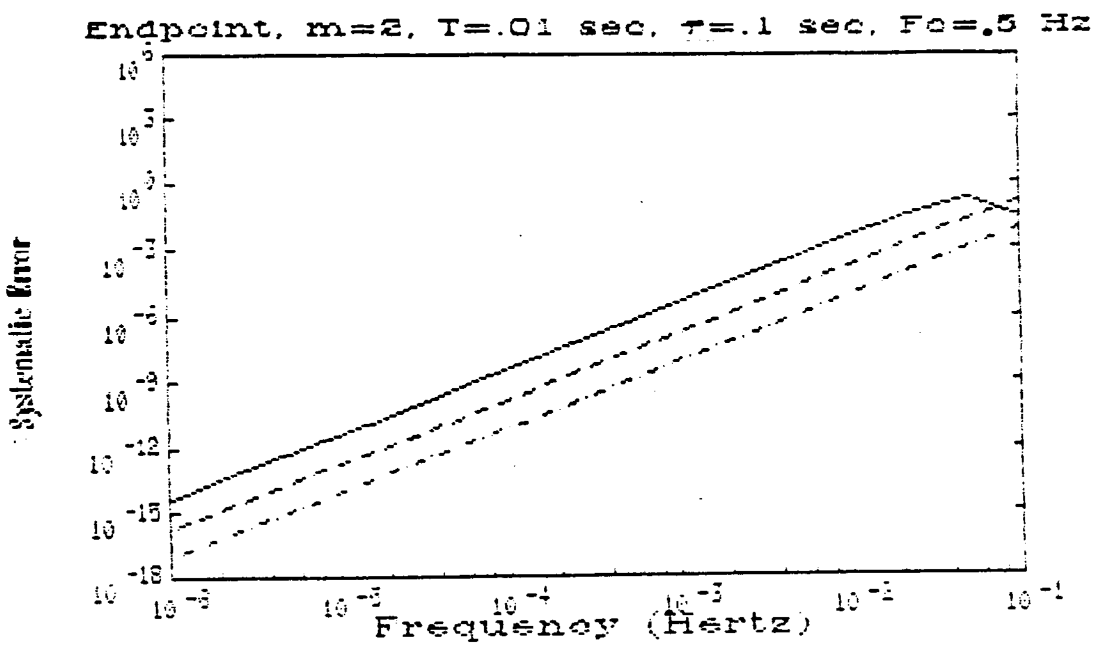
FIGURE 10
SCHEME D



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Linetype	R
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Dotdash	1.00

FIGURE 11
10 HERTZ PROCESSING, R = 0.01

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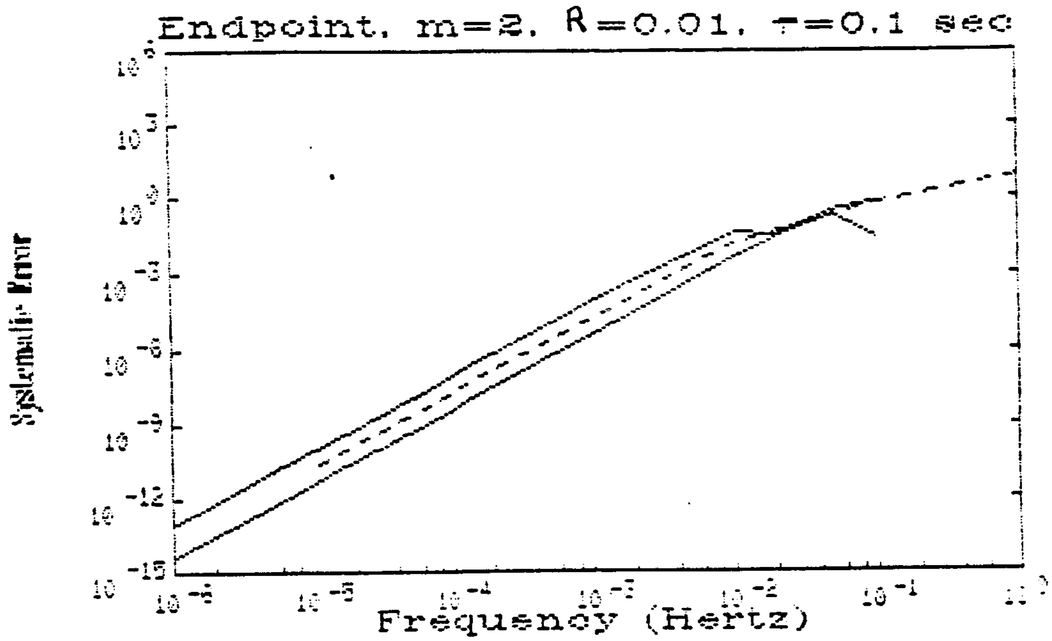
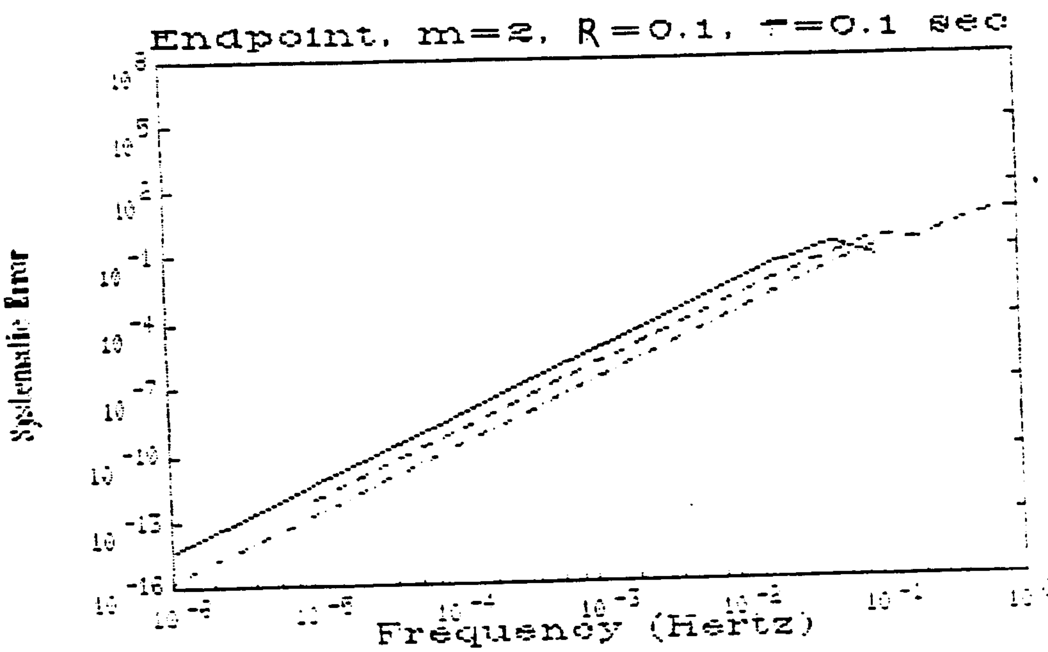


Legend

Linetype	Processing Scheme
Solid	No Bandlimiting
Dashed	A
Dotdash	D

FIGURE 12
10 HERTZ PROCESSING, R = 0.10



Legend

Linetype	Processing Scheme
Solid	No Bandlimiting
Dashed	A
Dotdash	D

FIGURE 13
10 HERTZ PROCESSING, R = 1.00

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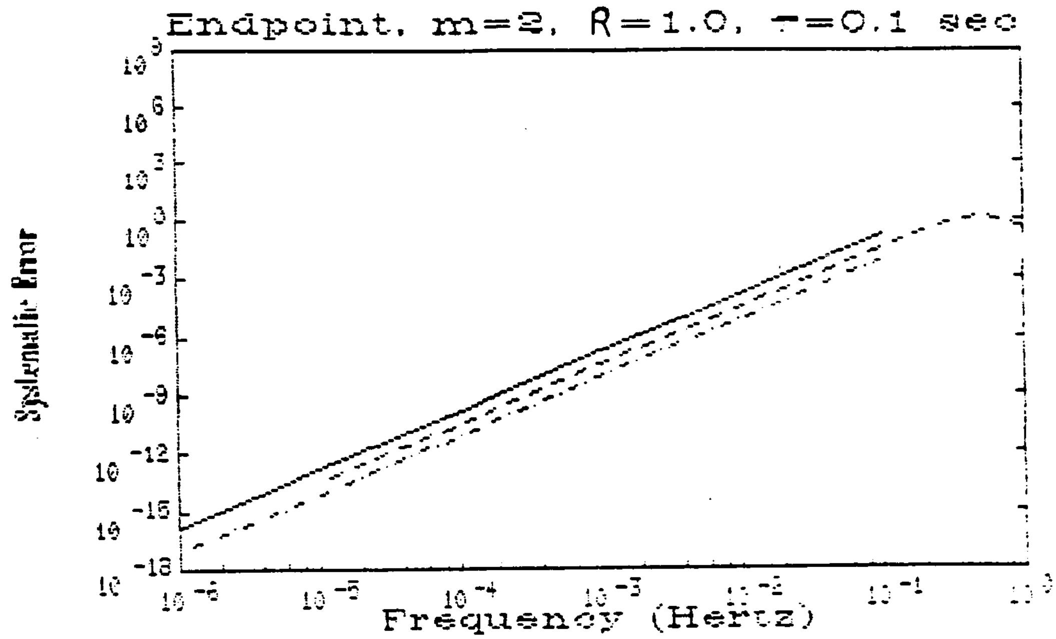

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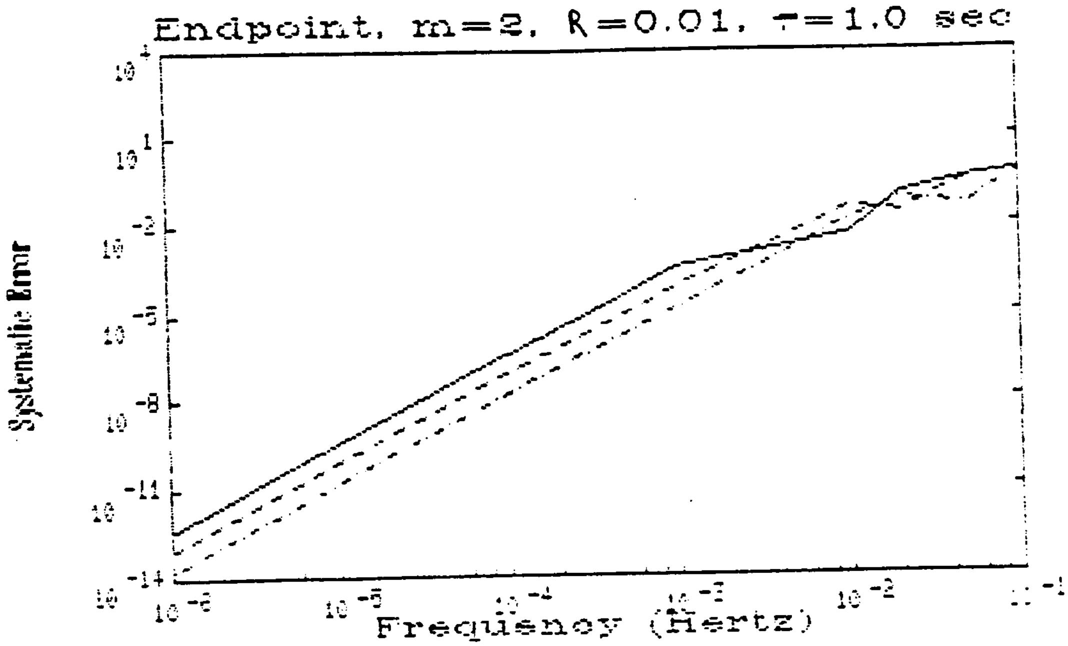
Linetype	Processing Scheme
Solid	No Bandlimiting
Dashed	A
Dotdash	D

FIGURE 14
1 HERTZ PROCESSING, R = 0.01



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Linetype	Processing Scheme
Solid	No Bandlimiting
Dashed	C
Dotdash	B

FIGURE 15
1 HERTZ PROCESSING, R = 0.10

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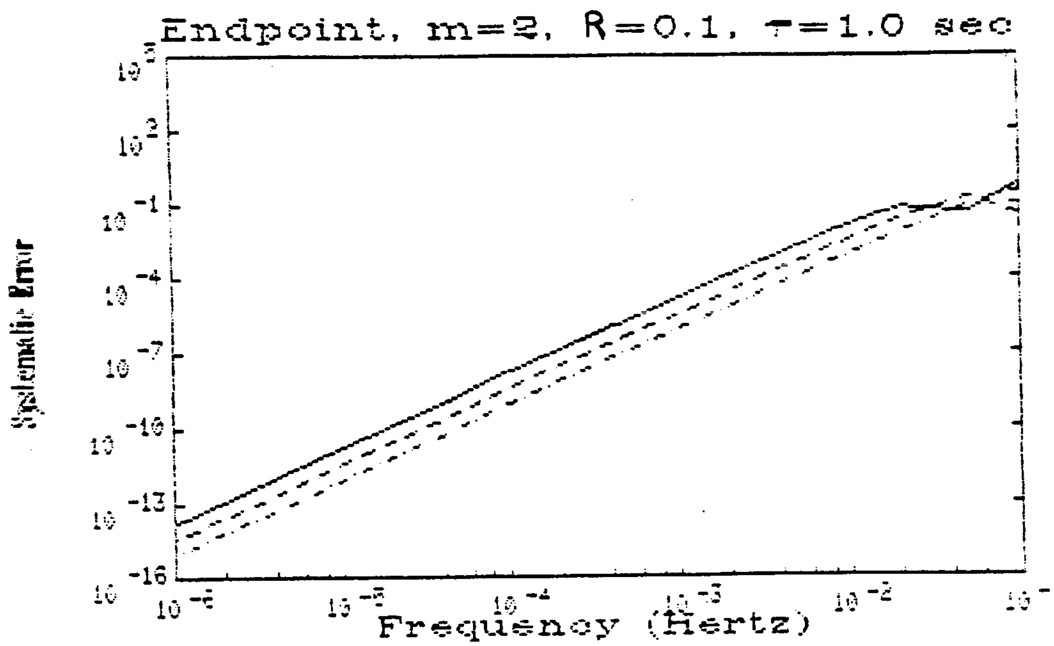

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Linetype	Processing Scheme
Solid	No Bandlimiting
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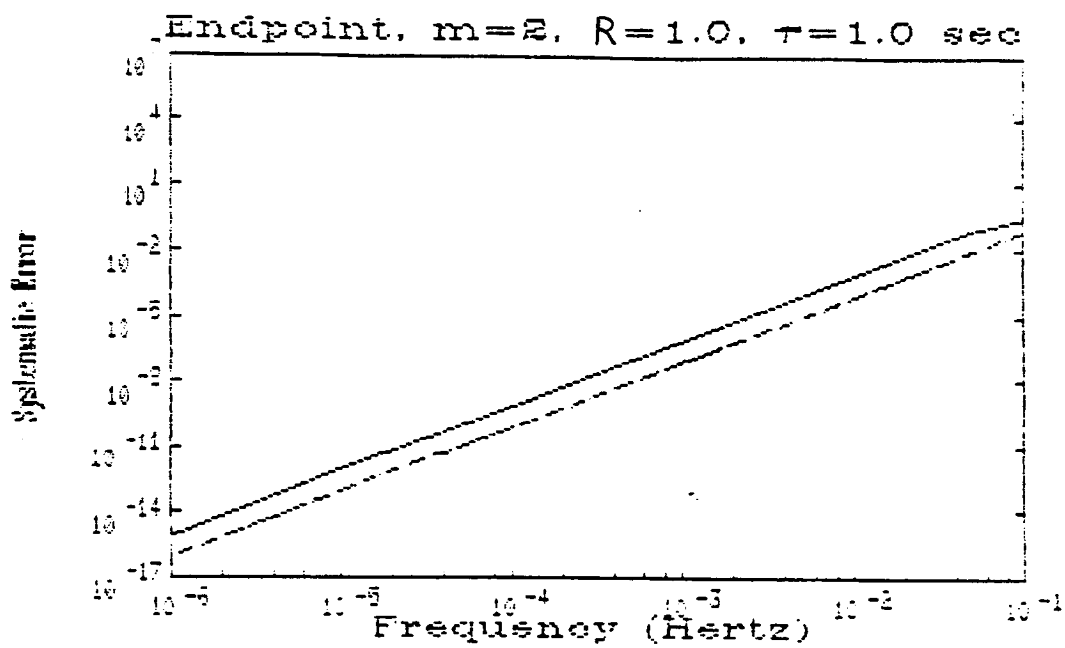
FIGURE 16
1 HERTZ PROCESSING, R = 1.00



Linetype	Legend Processing Scheme
Solid	No Bandlimiting
Dashed	C
Dotdash	B